List of Errata for the Book

"A SINGULAR Introduction to Commutative Algebra"

by G.-M. Greuel and G. Pfister

1. Mathematics

Page 84, line 5-7: should be replaced by:

Define $J_0 := J \cap K[y]$, and $I_0 := I \cap K[x]$. Then φ is induced by

$$\tilde{\varphi} : K[x]/I_0 \to K[y]/J_0\,, \quad x_i \mapsto f_i\,,$$

and we have a commutative diagram

Page 84, line 10+11: should be replaced by:

Solution: Let $J_0 = \langle g_1, \dots, g_s \rangle_{K[y]}$, $g_i \in K[y]$ and $I_0 = \langle h_1, \dots, h_t \rangle_{K[x]}$, $h_i \in K[x]$.[10] Set $H := \langle h_1, \dots, h_t, g_1, \dots, g_s, x_1 - f_1, \dots, x_n - f_n \rangle \subset K[x, y]$, and compute $H' := H \cap K[x]$ by eliminating $y_1, \dots, y_m$ from H (cf. Section 1.8.2).

Page 84, line -0: with the footnote

[10] Let $>$ be any monomial ordering on $K[x]$, let $f_1, \dots, f_m \in K[x]$, and let I be the ideal generated by $f_1, \dots, f_m$ in $K[x]_>$. Then (if $>$ is non-global) the inclusion $\langle f_1, \dots, f_m \rangle_{K[x]} \subset I_0 := I \cap K[x]$ may be strict. To compute I_0 in this case, one may proceed as follows: compute a primary decomposition $Q_1 \cap \dots \cap Q_s$ of $\langle f_1, \dots, f_m \rangle_{K[x]}$ (see Chapter 4). Assume that $Q_i K[x]_> \subsetneq K[x]_>$ iff $1 \le i \le t$. Then $I_0 = Q_1 \cap \dots \cap Q_t$.

Page 211, line 14: insert: "and if I is prime"

Page 220, line -2f: should be replaced by:

"p. 207]. The polynomial ring $K[x_1, \dots, x_n]$ and, more generally, each affine algebra $R = K[x_1, \dots, x_n]/I$ satisfy the following stronger condition: for each prime ideal $P \subset R$ and for each finite extension field L of $Q(R/P)$, the integral closure of R/P in L is a finite R/P-module. A Noetherian ring with this property is called *universally Japanese* (in honour of Nagata)."

Page 225, line 1: insert: Let $\overline{A}$ be finitely generated over A, and let $C =$ ...

Page 374, line -15: replace $\mathbb{Z}$-graded by $\mathbb{N}$-graded

2. Singular-Procedures

Page 157ff, Example 2.6.11: The procedure `extendedDiagonalForm` does not compute a diagonal form if the matrix does not have maximal rank. A corrected version of the procedure can be downloaded from

`http://www.singular.uni-kl.de/BOOK/singular.html`

3. Improvements

Page 46, resp. 118: For didactical reasons, it is better to use the following definition (on Page 46):

"(2) If $G = \{g_1, \ldots, g_s\}$, then f has a *standard representation* with respect to $\mathrm{NF}(- \mid G)$, that is,

$$f = \mathrm{NF}(f \mid G) + \sum_{i=1}^{s} a_i g_i, \quad a_i \in R, \quad s \geq 0,$$

satisfying $\mathrm{LM}(f) \geq \mathrm{LM}(a_i g_i)$ for all i such that $a_i g_i \neq 0$."

Page 46, line -7: the generators $h_1, \ldots, h_s$ can be omitted in the definition of N (without changing N)

Page 47, resp. 118, resp. 124: Similarly, e.g. (on Page 47):

"(2') for all $f \in R$ and $G \in \mathcal{G}$ there exists a unit $u \in R^*$ such that uf has a standard representation with respect to $\mathrm{NF}(- \mid G)$."

Page 221f, *Proof of Lemma 3.5.12*: should be replaced by:

Let L_0 be the splitting field of F, and let $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n \in L_0$ be the roots of F. Further, let B_0 be the integral closure of A in L_0. Then $\alpha_1, \ldots, \alpha_n \in B_0$ (since F is monic), and we have the following diagram

$$\begin{array}{ccccc} A & \subset & B & \subset & B_0 \\ \cap & & \cap & & \cap \\ Q(A) & \subset & L & \subset & L_0, \end{array}$$

where $L_0 \supset Q(A)$ is Galois. We consider the matrix

$$M := \begin{pmatrix} 1 & \alpha_1 & \ldots & \alpha_1^{n-1} \\ \vdots & \vdots & & \vdots \\ 1 & \alpha_n & \ldots & \alpha_n^{n-1} \end{pmatrix} \in \mathrm{Mat}(n \times n, B_0).$$

For $d := \det(M) = \pm \prod_{i<j}(\alpha_i - \alpha_j) \neq 0$ we have $d^2 \in Q(A)$, because d^2 is the discriminant of F.

Let $\mathrm{Gal}\big(L_0 \mid Q(A)\big) = \{\sigma_1, \ldots, \sigma_n\}$ with $\sigma_i(\alpha_1) = \alpha_i$. Let $b \in B_0$, then there exist $c_i \in Q(A)$ such that $b = \sum_{i=0}^{n-1} c_i \alpha_1^i$. We obtain

$$M \cdot \begin{pmatrix} c_0 \\ \vdots \\ c_{n-1} \end{pmatrix} = \begin{pmatrix} \sum_i c_i \alpha_1^i \\ \vdots \\ \sum_i c_i \alpha_n^i \end{pmatrix} = \begin{pmatrix} \sigma_1(b) \\ \vdots \\ \sigma_n(b) \end{pmatrix} \in B_0^n$$

and, consequently,

$$dM^{-1}M \cdot \begin{pmatrix} c_0 \\ \vdots \\ c_{n-1} \end{pmatrix} = \begin{pmatrix} dc_0 \\ \vdots \\ dc_{n-1} \end{pmatrix} \in B_0^n,$$

which yields $B \subset B_0 \subset (1/d^2) \cdot \sum_{i=0}^{n-1} A\alpha^i$, because $d^2 c_i \in B_0 \cap Q(A) = A$. This implies, in particular, that B is a finite A–module. □

Page 222, line -19: replace "$Q \in \mathrm{Ass}(PB)$" by "$PB \subset Q$"

Page 222, line -17ff: should be replaced by:

"*Proof.* We may assume $P = \langle 0 \rangle$ and use Corollary 3.3.3. □"

Page 251, line -2: insert: "To keep notations short in the following construction of the morphism φ, we identify the x_i (resp. the y_j) with their images in $K[y_1, \ldots, y_n]$ (resp. in $K[z_2, \ldots, z_n]$)."

Page 251, line 13: replace "as $y^2 - 1$ is reducible" by "since the minimal associated prime $\langle x^2 - 8, y - 1\rangle$ is not"

Page 310, line 14f: should be replaced by

"Indeed, if a linear combination satisfies $\sum_{i=1}^{a} c_i f_i \in Q^2$, then $\sum_{i=1}^{a} c_i \frac{\partial f_i}{\partial x_j} \in Q$, and $\mathrm{rank}\big(\frac{\partial f_i}{\partial x_j} \bmod Q\big)_{i,j \le a} = a$ implies $c_i = 0$ for all i."

4. Misprints

Page 39, line -17: omit the extra "|"
Page 41, line -13: replace "correspond" by "corresponds"
Page 45, line 17: replace "$\subset=$" by "$\subset$"
Page 51, line -13: replace "`x`" by "`xy`"
Page 69, line 6: should be replaced by

```
matrix(f)*L[3] - matrix(I)*L[1] - matrix(L[2]);   //test
```

Page 77, line -14: replace "n_i" by "n_1"
Page 80, line 13: should be replaced by:

$$I_1 : I_2 = \Big(I_1(K[x]_>)[t] : \langle h\rangle\Big) \cap K[x]_> \,.$$

Page 88, line 2: replace "in" by "of"
Page 95, line 22: replace "`contrahom`" by "`kontrahom`"
Page 96, line 15: replace "`cohom`" by "`kohom`"
Page 109, line 3: The chain should read: "$L :_M \langle x\rangle \subset L :_M \langle x^2\rangle \subset \cdots$"
Page 156, line 8: should read:
"$m_{c-n_i} \neq m_{c-n_i+1}$. Therefore, $\{n_1, \ldots, n_s\} = \{j \mid m_{c-j} \neq m_{c-j+1}\}$."
Page 162, line -5f: replace "`L[1][1]`" by "`L[2][1]`" (4 times)
Page 162, line -3: replace "`L[1][2]`" by "`L[2][2]`"
Page 162, line -1: replace "`L[2][1,1]`" by "`L[3][1,1]`"
Page 177, line 1: replace "`M[3]`" by "`M[2]`"
Page 177, line -9: omit "$\cap \bigoplus_{i=s+1}^{r} Re_i$"
Page 192, line 15ff: replace (4 times) "E_n" by "E_k"
Page 210, line -16: replace "$P \in \mathrm{Ass}(\langle 0\rangle)$" by "$P \in \mathrm{minAss}(\langle 0\rangle)$"
Page 212, line -4: replace "[46]" by "[51,124]"
Page 218, line -7: replace "surjective" by "dominant"
Page 219, line -2: replace the right "=" by "$\le$"
Page 228, line 13: replace "${}_{0\le i\le j\le s}$" by "${}_{1\le i\le j\le s}$"
Page 239, line -14: replace "$I \subset A$" by "$I \subsetneq A$"
Page 241, line 1: replace "$I \subset A$" by "$I \subsetneq A$"
Page 247, line 9: replace "R" by "$K[x]$"
Page 248, line 13: replace "$\langle x_1 - g_1, \ldots, x_{n-1} - g_{n-1}, g_n\rangle \subset K[x]$" by "$\langle x_1 + g_1, \ldots, x_{n-1} + g_{n-1}, g_n\rangle \subset K[x]$"
Page 256, line 6: replace "$\langle I, h^n\rangle$" by "$\langle I, h\rangle$"
Page 258, line 17: replace "I" by "J"
Page 275, line -5: replace "graded K-algebra" by "Noetherian graded K-algebra"

Page 275, line -4: replace "graded A-module" by "finitely generated graded A-module"
Page 277, line 1: replace "graded K-algebra" by "Noetherian graded K-algebra"
Page 279, line -2: replace "$Q(t) \in K[t]$" by "$Q(t) \in \mathbb{Z}[t]$"
Page 282, line 3: replace "$Q(t) \in K[t]$" by "$Q(t) \in \mathbb{Z}[t]$"
Page 282, line 20: replace "bases" by "basis"
Page 283, lines 13,19,21: replace (3 times) "graded K-algebra" by "Noetherian graded K-algebra"
Page 284, line 8: replace "graded K-algebra" by "Noetherian graded K-algebra"
Page 296, line 9: replace "$\mathrm{Gr}_m(A)$" by "$\mathrm{Gr}_{\mathfrak{m}}(A)$"
Page 312, line 1: replace "be a local ring" by "be a regular local ring"
Page 312, line 1: replace "R/P" by "A/P"
Page 339, line -13: insert (2 times) "$, A/\langle x\rangle)$"
Page 339, line -6: insert (2 times) "$, A/\langle x\rangle)$"
Page 349, line 4: replace "Let A be a ring" by "Let A be a Noetherian ring"
Page 350, line 22: replace "$\mathrm{Spec}(A/I_j)$" by "$\mathrm{Spec}(A/I_{j-1})$"
Page 356, line 14: replace "$w = (w_1, \ldots, w_n) \in \mathbb{Z}^n$" by "$w = (w_1, \ldots, w_n) \in (\mathbb{Z} \setminus \{0\})^n$"
Page 358, line 10: replace "...$M \overset{\pi}{(} A/\mathfrak{m}^n)$..." by "...$M \xrightarrow{\pi} (A/\mathfrak{m}^n)$..."
Page 359, line -13: replace "$A[x]_{\langle x\rangle}$" by "$A[x]_{\langle \mathfrak{m},x\rangle}$"
Page 390, line 12: replace "(B/I)" by "A"
Page 397, line -2: replace "[46]" by "[51]"
Page 398, line -1: replace "[98]" by "[124]"
Page 408, line 9: insert "for I a prime ideal"
Page 443, line -5: replace "$\mathbb{P}^n_K$" by "$\mathbb{P}^n_R$"
Page 484, line -17: replace "7.4.7" by "7.4.6"
Page 544, lines 5,6: replace "gegree" by "degree"
Page 557, line 10: replace "Aglebraic" by "Algebraic"
Page 559, line -10: replace "Cambrigde" by "Cambridge"
Page 559, line -1: replace "Elemtare" by "Elementare"
Page 560, line 26: replace "at" by "of"